RUTGERS
Graduate School of Education

Learning and Teaching
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Topics in Mathematics Education: Critical Thinking and Reasoning
15:254:551
3 Credits

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Office Hours: by appointment	Prerequisites or other limitations: none
Mode of Instruction: ___ Lecture ___ Seminar ___ Hybrid __X_ Online ___ Other	Permission required: _x_ No __Yes Directions about where to get permission numbers: from the instructor

Learning goals

1. To learn about how children reason about fraction ideas.
2. To provoke consideration of the way that fractions are introduced in the elementary grades (and its potential impact on instruction in the secondary grades) through examining the critical thinking and reasoning in students by studying video clips.
3. To consider the relationship between learning fractions and learning other topics in mathematics and the implications of meaningful mathematical learning for teacher education and professional development.
4. To represent your own understanding of children's knowledge of fractions by building multimedia narratives that focus on specific concept(s) within the broader topic.

Course Description

The purpose of this course is to learn about how children reason about fraction ideas. You will be introduced to literature on the obstacles to fraction learning and asked to examine, in detail, how some students build understanding of fundamental fraction concepts and operations. You will be given access to the Rutgers Video Mosaic Collaborative (VMC) Repository collection that stores video and related data from studies of children's problem solving with fractions. The course also is intended to provoke your consideration of the way that fractions are introduced in the elementary and secondary grades by examining the critical thinking and reasoning in students by studying video clips. The means for doing this will be through readings, working on strands of fraction problems, and examining the learning of fraction ideas as a sense-making activity. You will be asked to consider the relationship, if any, between learning fractions and learning other topics in mathematics and the implications of meaningful mathematical learning for teacher education and professional development. Finally, you will use the resources and tools of the VMC to build multimedia artifacts (VMCAnalytics) that focus attention on specific concepts of fractions learning and illustrates your key point(s) through selection of video events and composition of accompanying text descriptions.

Class materials:

You will need to acquire an Introductory Set of Cuisenaire Rods (contains 74 rods) for use in explorations and model building. If you have access to a set from the school or district where you work, then you could borrow the materials. However, they are not expensive. The set is available for purchase via the Internet or in certain retail stores for about $15-20. Wooden rods are recommended

but plastic rods are acceptable. iPad users might wish to purchase the vBlocks app for $2.99 for using virtual manipulatives (it includes both Cuisenaire rods and Unifix cubes). Electronic images of Cuisenaire rod models could be saved to Pages, then exported/uploaded to share with classmates in this online course. Digital images of physical rod models also could be shared in this manner. The reading assignments and access to VMC videos will be made available on our eCollege course site.

Course Requirements

Successful completion of the course requires that actively engage in all activities and submit all assignments. This process requires that you:

1. Complete survey at the beginning and a brief reflection (1-2 pages) at the end of the course, each by its specified due date as indicated on the eCollege course site. These are required but ungraded assignments intended to inform instructional design and course improvement.
2. Compose and post individual written assignments, which will include some problem solving activities and consist primarily of reflections on, reactions to, and analyses of the videos and readings. These activities need to occur towards the first part of the course unit to enable discussion of others' ideas during the latter portion. Refer to our eCollege site for specific due dates. Also refer to General Guidelines for Online Discussions (see section below).
3. Engage in threaded group discussions by composing and posting responses to the postings that have been made by your group members in the earlier portion of the unit. Your original posting should be made prior to your responding posts to the contribution of others. Refer to General Guidelines for Online Discussions (see section below).
4. You will become acquainted with the video clips from the fraction strand, which have been cataloged for access via the VMC repository. Each video resource has a persistent URL and is linked to a wealth of metadata (info about the video). You will be expected to know how to search to find videos within the main VMC site and the VMCAnalytic tool workspace.
5. You will be introduced to the VMCAnalytic tool, which you will use to select and annotate video events (i.e., shorter video segments) to build your own Analytics. They are multimedia artifacts that tell a story about mathematical learning and teaching. Each VMCAnalytic is a new artifact of research that potentially can be published as a new resource on the VMC. Through research we have identified criteria for assessing quality of VMCAnalytics, and the rubric we have used for scoring them will be shared with you as part of the introduction.
6. You will produce three VMCAnalytics: the first will analyze a single video clip (2-3 events); the second will analyze video from two clips (3-4 events); and the third will analyze learning over a period of time using several video clips as resources (5-8 events). You will prepare a written narrative that accompanies each of your Analytics, with each paper having a length corresponding to what is appropriate for situating the story you tell (e.g., 0.5 page, 1 page, 2-3 pages). The assignments will be staggered to provide you with feedback for improving your work, with first due about mid-way, second about two-thirds through, and third at end. Specific due dates will be posted on the eCollege site.

Grading Criteria

Your grade for the course will be determined by the work that you produce towards meeting the course requirements described in the numbered points above. Required course work contributes to a final grade based on the following percentages:

Requirement 1 (surveys and final reflections):	ungraded yet required for course completion
Requirements 2-3 (eCollege discussion):	65% of grade
Requirement 6 (three VMCAnalytics):	35% of grade (5%, 10%, and 20%, respectively)

ALL students are expected to contribute regularly to the course. You will be evaluated on the following:

(1) The quality and frequency of your postings from the problem solving, readings, and analyses of video. This means that you are expected to initiate discussions in each unit as well as respond to others' postings. *Your grade will be lowered* if you are not actively engaged in postings on a regular basis each unit;
(2) The VMCAnalytics and accompanying papers you will create that deal with students' fraction learning and may include implications for teaching. These will be evaluated on your thoughtfulness and insights into the learning process and how students are building particular fraction knowledge based on the events you select and annotate with your text descriptions to tell a coherent story. A research-based scoring rubric guides the evaluation.

Course Outline And Assignments

Unit 1 Jan 21 – 27	**Individual Work**: Complete the survey first and submit it to your dropbox; then do problem solving tasks and assigned readings. Review introductory materials about a particular way of using Cuisenaire rods and do the problem-solving activities that are posted. Share representations of your rod models to support solutions as part of the discussion. **Assigned readings**: (**1**) Davis, G., Hunting, R. P. & Pearn, C. (1993). What might fractions mean to a child and how would a teacher know?" *The Journal of Mathematical Behavior, 12*(1), 63-76. (**2**) Maher, C. A. & Alston, A. (1989). Is meaning connected to symbols? An interview with Ling Chen. *The Journal of Mathematical Behavior, 8*(3), 241-248. **Contribute to online discussion** per course guidelines and the prompts that are posted online in the introduction to and topics for this unit.
Unit 2 Jan 28 – Feb 3	**Study video clips:** **(1)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067409 VM Title: *Fraction as number, an introduction, Clip 1 of 8: Assigning number names to rods* (**2**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067417 VM Title: *Fraction as number, an introduction, Clip 2 of 8: Students model fraction problems* (**3**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067411 VM Title: *Fraction as number, an introduction, Clip 3 of 8: Permanent color names and flexible number names for rods* (**4**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067412 VM Title: *Fraction as number, an introduction, Clip 4 of 8: Establishing a unit*

	for comparing length of rods (**5**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067414 VM Title: *Fraction as number, an introduction, Clip 6 of 8: Problem posing for the class* (**6**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067416 VM Title: *Fraction as number, an introduction, Clip 8 of 8: Emphasizing precise language in problem posing* **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 3 Feb 4 – 10	**Assigned reading:** Maher, C. A. & Davis, R. B. (1995). Children's explorations leading to proof. In C. Hoyles and L. Healy (eds.), *Justifying and proving in school mathematics*, pp. 87-105. London: Mathematical Sciences Group, Institute of Education, University of London **Study video clips:** **(1)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054465 VM Title: *Fractions, Grade 4, Clip 1 of 4: David's upper and lower bound argument* **(2)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054749 VM Title: *Fractions, Grade 4, Clip 2 of 4: Additive vs. multiplicative reasoning* **(3)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054751 VM Title: *Fractions, Grade 4, Clip 4 of 4: Designing a new rod set* **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 4 Feb 11 – 17	**Study video clips:** **(1)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067421 VM title: *Reviewing rod relationships and the candy bar problem, Clip 2 of 6: What is the number name for red when the yellow and light green rod is two? Brian and Jacquelyn* **(2)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067422 VM title: *Reviewing rod relationships and the candy bar problem, Clip 3 of 6: What is the number name for red when the yellow and light green rod is two? A whole class discussion* **(3)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067423 VM title: *Reviewing rod relationships and the candy bar problem, Clip 4 of 6: Switching units, candy bar metaphor* **(4)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067424 VM title: *Reviewing rod relationships and the candy bar problem, Clip 5 of 6: Comparing one half and one third, part 1*
Unit 5 Feb 18 – 24	**Assigned reading:** Steencken, E. P. & Maher, C. A. (2003). Tracing fourth graders' learning of fractions: Episodes from a yearlong teaching experiment. *The Journal of Mathematical Behavior, 22* (2), 113-132. **Study video clips:**

	(1) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059681 VM title: *Introducing Fraction Equivalence and an Exploration of Fraction Comparison, Clip 1 of 4: Equivalent fractions, a debate* **(2) URL:** http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059685 VM title: *Introducing Fraction Equivalence and an Exploration of Fraction Comparison, Clip 2 of 4: An introduction to proportional reasoning.* **(3) URL**: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059691 VM title: *Introducing fraction equivalence and an exploration of fraction comparison, Clip 3 of 4: Proportional Reasoning Continued* **(4) URL:** http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059695 VM title: *Introducing fraction equivalence and an exploration of fraction comparison, Clip 4 of 4: Finding the number name for the difference between one half and one third* **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 6 Feb 25 – Mar 3	**Class Project:** Introduction to making VMCAnalytics via instructional video **Assigned Reading:** Agnew, G., Mills, C. M., & Maher, C. A. (2010). VMCAnalytic: Developing a collaborative video analysis tool for education faculty and practicing educators. In R. H. Sprague, Jr. (Ed.), *Proceedings of the 43rd Annual Hawaii International Conference on System Sciences (HICCS-43): Abstracts and CD-ROM of Full Papers*. IEEE Computer Society, Conference Publishing Services: Los Alamitos, CA. **Examine** scoring rubric and sample VMCAnalytics **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 7 March 4 – 10	**Study video clips:** **(1)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067427 VM title: *Comparing fractions, a whole class debate, Clip 2 of 5: Remembering the candy bar* **(2)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000068675 VM title: *Comparing fractions, a whole class debate, Clip 3 of 5: Brian challenges the girls' argument* **(3)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067429 VM title: *Comparing fractions, a whole class debate, Clip 4 of 5: An argument based on a different model* **(4)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067430 VM title: *Comparing fractions, a whole class debate, Clip 5 of 5: The difference is one sixth* **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 8 March 11 – 17	**Individual Work**: Submit your first VMCAnalytic that analyzes a single video clip and the short accompanying paper (0.5 page). See due date on course site. **Study video clips:**

	(1) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067447 VM title: *Discovering equivalent fractions and introducing fraction notation, Clip 1 of 5: Boats and fish, a conversation about building models* **(2)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067448 VM title: *Discovering equivalent fractions and introducing fraction notation, Clip 2 of 5: David and Meredith compare one half and two thirds* **(3)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067449 VM title: *Discovering equivalent fractions and introducing fraction notation, Clip 3 of 5: Alan and Erik compare one half and two thirds* **(4)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067450 VM title: *Discovering equivalent fractions and introducing fraction notation, Clip 4 of 5: Danielle and Gregory compare one half and two thirds* **(5)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067451 VM title: *Discovering equivalent fractions and introducing fraction notation, Clip 5 of 5: Compare one half and two thirds, establishing equivalence* **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 9 March 25 – 31	**Assigned readings:** (1) Alston, A. S., Davis, R. B., Maher, C. A., & Martino, A. M. (1994). Children's use of alternative structures. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (2)*, 248-255. Lisboa, Portugal: University of Lisboa. (2) Maher, C. A., Martino, A. & Davis, R. B., (1994). Children's different ways of thinking about fractions. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (3)*, 208-215. Lisboa, Portugal: University of Lisboa. **Study video clips:** **(1)** URL:http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000055290 VM Title: *Fraction problems, Sharing and Number Lines, Clip 3 of 5: Comparing unit fractions* **(2)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000055291 VM title: *Fraction problems, Sharing and Number Lines, Clip 4 of 5:Jessica's and Andrew's number line* **(3)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000055292 VM Title: *Fraction problems, Sharing and Number Lines, Clip 5 of 5: Placing fractions on the number line* **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 10 April 1 – 7	**Individual Work**: Prepare your second VMCAnalytic analyzing two video clips and the short accompanying paper (1 page). See due date on course site. **Study video clips:** **(1)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067471 VM title: *The infinite number line, Clip 1 of 4: Naming points on the number*

	line **(2)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067472 VM title: *The infinite number line, Clip 2 of 4: Placing integers on the number line* **(3)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067473 VM title: *The infinite number line, Clip 3 of 4: How many numbers between 0 and 1?* **(4)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067474 VM title: *The infinite number line, Clip 4 of 4: Placing fractions and mixed numbers on the number line* **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 11 April 8 – 14	**Assigned reading:** Schmeelk, S. & Alston, A. (2010). From equivalence to rational numbers: The case of Meredith. In P. Brosnan, D. B. Erchick & L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (6),* 720-727. Columbus, OH: The Ohio State University **Study video clips:** **(1)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059707 VM title: *Number Line Models and Placing Numbers on the Big Number Line, Clip 1 of 4: Meredith's Number Line Models* **(2)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059708 VM title: *Number Line Models and Placing Numbers on the Big Number Line, Clip 2 of 4: Discussion of Meredith's Number Line Models* **(3)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059711 VM title: *Number Line Models and Placing Numbers on the Big Number Line, Clip 3 of 4: Equivalent names for the number named one* **(4)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059709 VM title: *Number Line Models and Placing Numbers on the Big Number Line, Clip 4 of 4* **Contribute to online discussion** per course guidelines and prompts in this unit.
Unit 12 April 15 – 21	**Assigned reading**: Steffe, L. P. (2010). Perspectives on children's fraction knowledge. In L. P. Steffe and J. Olive, *Children's fractional knowledge*, pp. 12-25. New York: Springer. **Study video clips:** **(1)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000067475 VM title: *The number line and equivalent fractions, Clip 1 of 1: Multiple representations for equivalent fractions* **Contribute to online discussion** per course guidelines and the prompts that are posted online in the introduction to and topics for this unit.
Unit 13	**Study video clips:**

April 22 – 28	**(1)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000057790 VM title: *Division of Fractions, Clip 1 of 3: How Many One Twelfths are in One?* **(2)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000057791 VM title: *Division of Fractions, Clip 2 of 3: Meredith Writes a Number Sentence* **(3)** URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000057792 VM title: *Division of Fractions, Clip 3 of 3: A Whole Class Discussion About Number Sentences* **Contribute to online discussion** per course guidelines and the prompts that are posted online in the introduction to and topics for this unit.
Unit 14 Apr 29 – May 5	**Individual Work**: (**1**) Prepare a short (1-2 page) reflection on this course. Guidelines to be posted in the introduction to this course unit. Submit your completed work by uploading file to your dropbox for that assignment on our eCollege course site. (**2**) Submit your final project of VMCAnalytic and accompanying paper. Details of assignment and submission process are posted on the eCollege course site.

Required Readings

Agnew, G., Mills, C. M., & Maher, C. A. (2010). VMCAnalytic: Developing a collaborative video analysis tool for education faculty and practicing educators. In R. H. Sprague, Jr. (Ed.), *Proceedings of the 43rd Annual Hawaii International Conference on System Sciences (HICCS-43): Abstracts and CD-ROM of Full Papers.* IEEE Computer Society, Conference Publishing Services: Los Alamitos, CA.

Alston, A. S., Davis, R. B., Maher, C. A., & Martino, A. M. (1994). Children's use of alternative structures. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (2)*, 248-255. Lisboa, Portugal: University of Lisboa.

Davis, G., Hunting, R. P. & Pearn, C. (1993). What might fractions mean to a child and how would a teacher know?" *The Journal of Mathematical Behavior, 12*(1), 63-76.

Maher, C. A. & Alston, A. (1989). Is meaning connected to symbols? An interview with Ling Chen. *The Journal of Mathematical Behavior, 8*(3), 241-248.

Maher, C. A. & Davis, R. B. (1995). Children's explorations leading to proof. In C. Hoyles and L. Healy (eds.), *Justifying and proving in school mathematics*, pp. 87-105. London: Mathematical Sciences Group, Institute of Education, University of London.

Maher, C. A., Martino, A. & Davis, R. B., (1994). Children's different ways of thinking about fractions. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (3)*, 208-215. Lisboa, Portugal: University of Lisboa.

Schmeelk, S. & Alston, A. (2010). From equivalence to rational numbers: The case of Meredith. In P. Brosnan, D. B. Erchick & L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (6),* 720-727. Columbus, OH: The Ohio State University.

Steencken, E. P. & Maher, C. A. (2003). Tracing fourth graders' learning of fractions: Episodes from a yearlong teaching experiment. *The Journal of Mathematical Behavior, 22* (2), 113-132.

Steffe, L. P. (2010). Perspectives on children's fraction knowledge. In L. P. Steffe and J. Olive, *Children's fractional knowledge*, pp. 12-25. New York: Springer.

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Make sure that you provide proper citations for all materials that you use in your project work.